EquiMus: Energy-Equivalent Dynamic Modeling and Simulation of Musculoskeletal Robots Driven by Linear Elastic Actuators

EquiMus：基于能量等效的肌肉骨骼机器人动力学建模与仿真算法

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1. 课题背景与意义 | 刚柔耦合仿生机器人研究

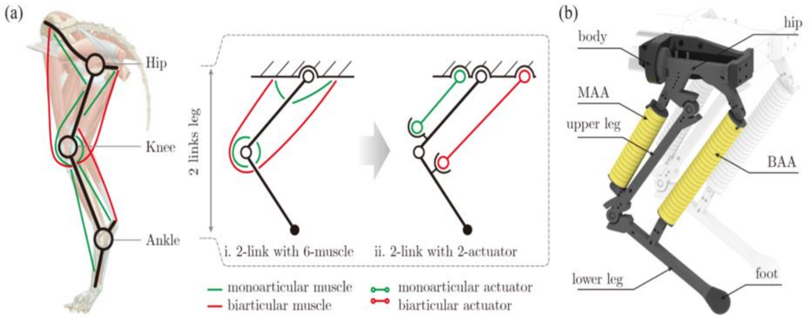
肌肉骨骼机器人：刚柔并济

- 刚性骨架结合柔性驱动器，为机器人提供高负载能力与环境适应性
- 弹性驱动器（Elastic Actuators, EA）具有高能量密度和固有的柔顺性

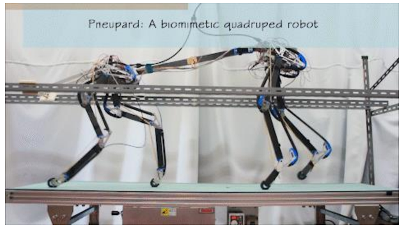
模型与控制：主要采用准静态/开环控制。但对于可控柔顺性与高爆发运动，**动力学模型**指导机器人机构设计、控制策略设计等。动力学建模中，需要考虑**驱动器的变质量分布。**

肌肉骨骼机器人

驱动器占比490.7g/999.6g



Dong, et al, 2025



K.S. Aschenbeck, et al, 2006

M. T. Tolley, et al.2021

Niiyama et al. 2007

地形适应能力、灵活性与安全性——刚柔协同的肌肉骨骼系统仿生范式

准静态/开环控制

Leg jumping

Video S.1.

Key Points

- FEA-powered bionic leg that moves, feels, and reacts like muscle. Bridging artificial actuation and biological performance.
- System showed biological-like behaviors: compliant, nonlinear, and strongly coupled
- Initial goal: make the leg jump — simple in idea, complex in dynamics
- **Simulation attempts failed**: standard rigid-body engines couldn’t capture soft behavior

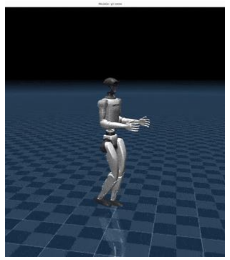
→ Triggered the central question: How can softness be simulated with physical consistency?

Dong, et al. IEEE Transactions on Robotics (2025).

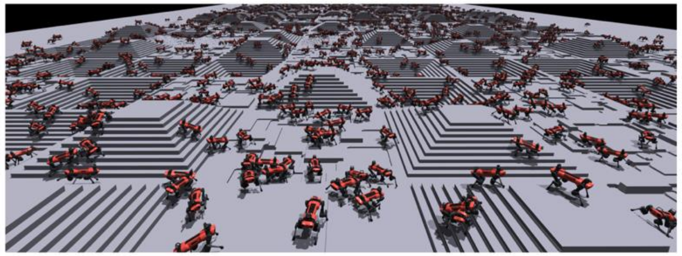
1. 课题背景与意义 | 刚柔耦合仿生机器人研究

刚体机器人的模型-仿真范式

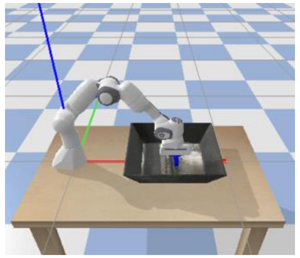
- 理论基础：多刚体动力学
- 软件基础：有丰富的开源动力学仿真引擎，提供多样的测试环境和数据驱动控制方法的接口，大幅降低了模型理论推导的复杂度及数据获取的成本



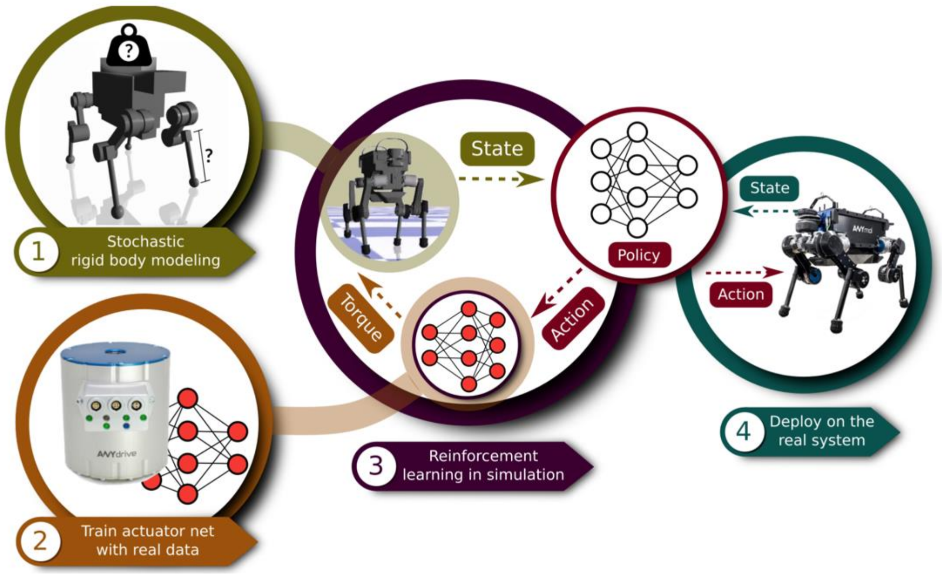
UnitreeRLGym
开源机器人模型



Legged Gym, Nvidia
高度并行化仿真



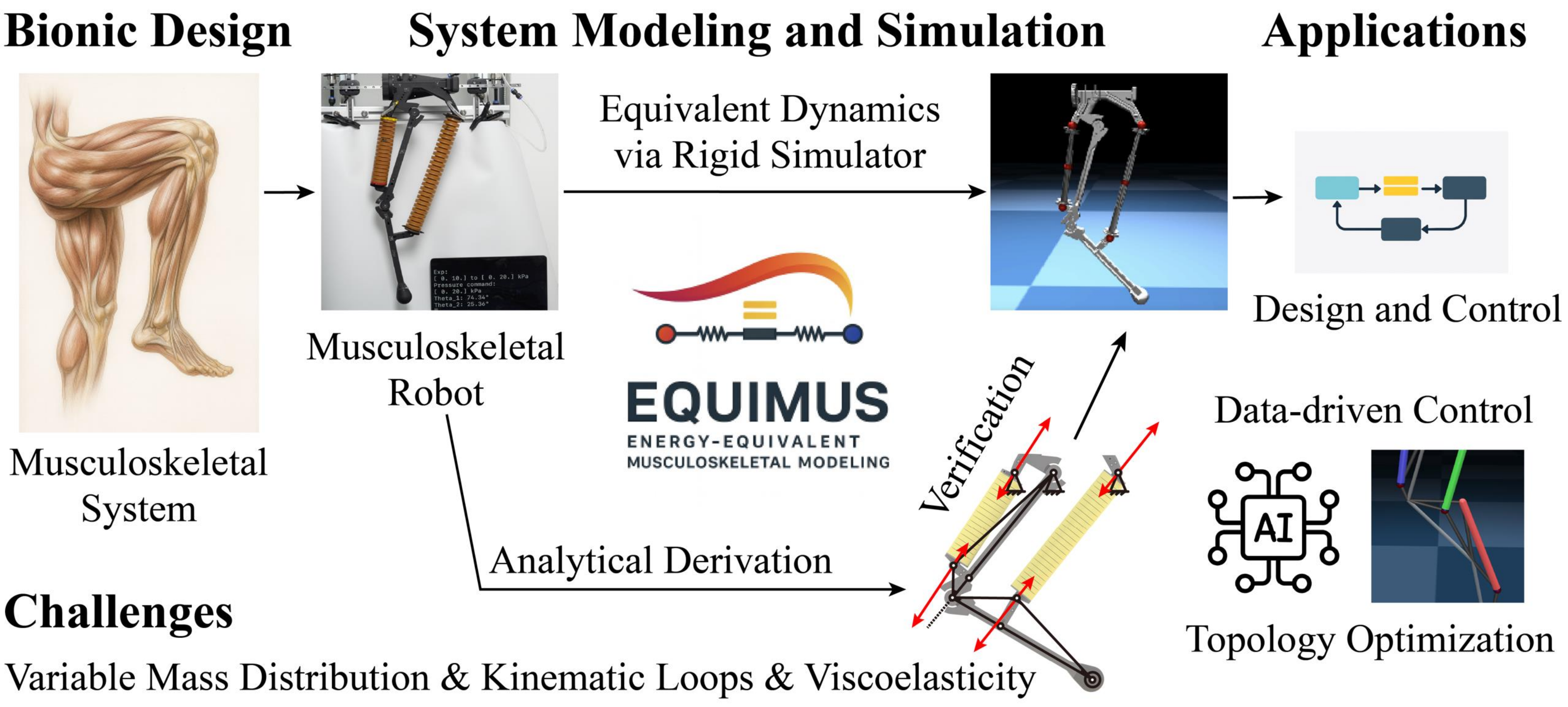
Pybullet
适用于机器学习



HWANGBO, et al, 2019

- 刚性机器人的运动控制研究形成了仿真-模型控制/强化学习-sim2real部署的**研究范式，自动计算模型，数据采集效率高**

1. 课题背景与意义 | 跨越柔性仿生设计-具身智能的 Gap



如何高效地建立系统的动力学模型和仿真，并具备和刚体仿真器结合的潜力，以实现未来基于数据驱动的结构设计优化、控制策略探索

2. 文献综述| 软体机器人的动力学建模方法

连续介质力学模型
Continuum mechanics models

由物理规律，如 Cosserat 梁理论和非线性 Euler-Bernoulli 梁模型得到连续、无限维构型空间描述

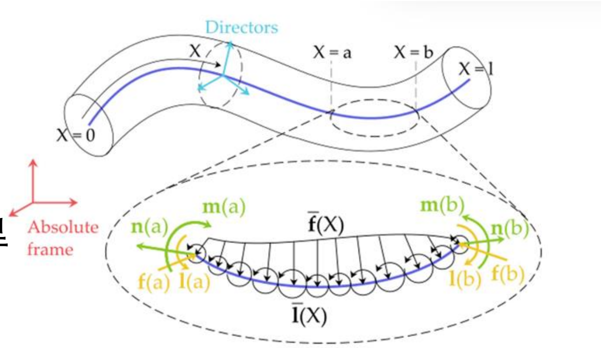
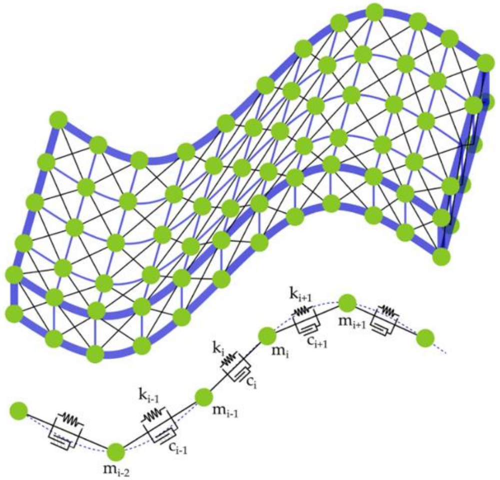


Fig. 3. Cosserat rod representation.

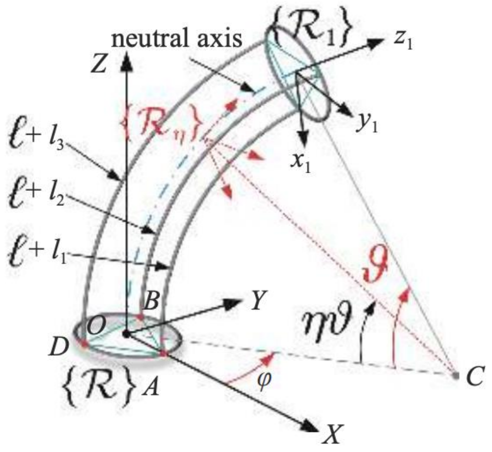
离散材料模型
Discrete material models

采用预定义的有限维构型空间来建模，**适用于减少参数，简化模型**



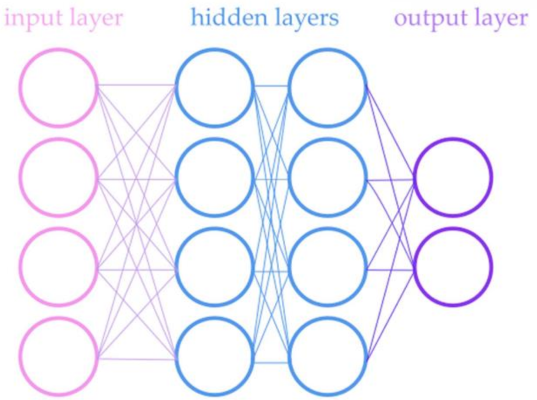
几何模型
Geometrical models

假设机器人具有特定的形状和变形方式，例如分段常曲率模型（Piecewise-Constant Curvature, PCC）



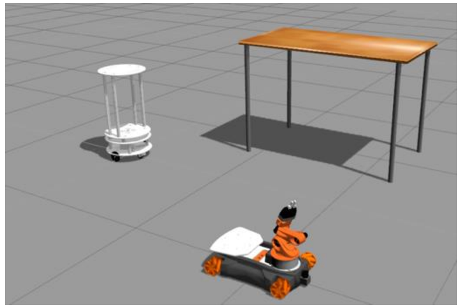
代理模型
Surrogate models

使用神经网络（NN）和机器学习（ML）对致动器和机器人建模，依赖高效的仿真或实验平台进行数据采样。



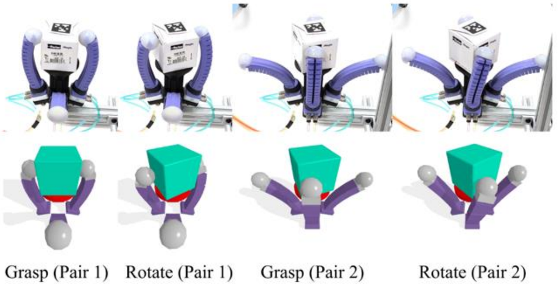
2. 文献综述| 典型的刚性/柔性机器人动力学仿真方法

□ 综合考虑仿真器模型复杂度、运动学闭环支持、刚柔耦合支持



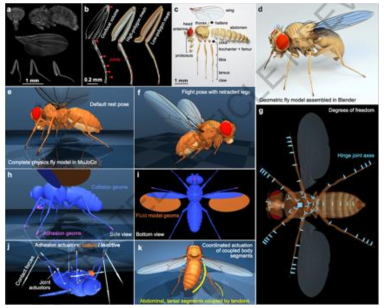
Gazebo
Nathan Koenig, et al, 2004

纯刚体，和ROS结合良好



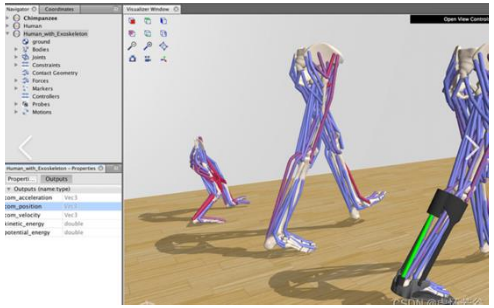
SoMo
Graule, et al, 2021

Pybullet结合梁理论，解决刚柔耦合的准静态操作问题



With MuJoCo
Vaxenburg, et al, 2025

MuJoCo结合空气动力学实现仿生飞行



OpenSim
Scott, et al, 2007

生物力学分析，采用无质量肌肉模型

MyoSim
Wang, et al, 2022

基于MuJoCo+RL构建人体理想肌肉骨骼

多种仿真器调研对比：MuJoCo仿真器可以较好支持并联结构，但忽略了驱动器的质量

TABLE I
COMPARISON OF DIFFERENT SIMULATION FRAMEWORKS

Framework	Modeling Approach	Physics Complexity	Kinematic Loop	R–S Hybrid Robots	DMR
PyBullet/Gazebo	rigid-body	low	×	×	×
Webots	rigid-body	low	×	×	×
Elastica	Cosserat rods	medium	×	×	✓
SOFA	FEM, finite element method	high	✓	✓	✓
SoMo/SoMoGym	rigid-body	low	×	✓	✓
SoftManiSim	rigid-body + Cosserat	medium	×	✓	✓
OpenSim	rigid-body + muscle model	medium-high	✓	✓	×
MuJoCo/MyoSim	rigid-body + muscle model	medium	✓	✓	×
EquiMus*	**rigid-body equivalence**	**low**	**✓**	**✓**	**✓**

R–S = rigid–soft; DMR = dynamic mass redistribution (in this work).

💡 What Problems Does EquiMus Solve?

EquiMus addresses four long-standing pain points in musculoskeletal robot simulation:

1. Soft actuators are difficult to model—especially with dynamic mass redistribution

Most simulators approximate muscles as massless springs, leading to incorrect dynamics.
→ EquiMus preserves actuator inertia, elasticity, damping, and work through an energy-equivalent formulation.

2. Musculoskeletal systems contain kinematic loops

URDF/PyBullet cannot represent multi-joint muscles or loop closures.
→ EquiMus uses MJCF with automatically generated constraints to model loops safely and robustly.

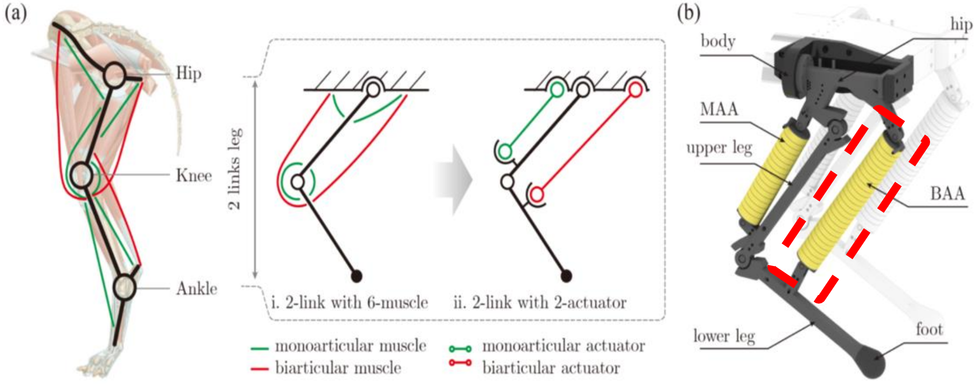
3. High-fidelity soft-body simulation is too slow for control & RL

FEM and Cosserat rod models are accurate but far from real time.
→ EquiMus achieves MuJoCo-level performance (>140× real-time) while retaining physical consistency.

4. No unified workflow from CAD → simulation → real robot

Current workflows require hand-built hacks and produce inconsistent dynamics.
→ EquiMus provides a clean, reproducible end-to-end pipeline, including calibration and sim-to-real examples.

3. 研究内容| 离散化模型及其能量等效分析



基于直线型弹性体的刚柔耦合机器人系统
（以实验室研究的单足机器人为例）

基本假设

- 仅在轴向方向发生均匀形变
- 驱动力沿轴向方向作用
- 近似等效为弹簧-质量-阻尼模型（三元素模型）

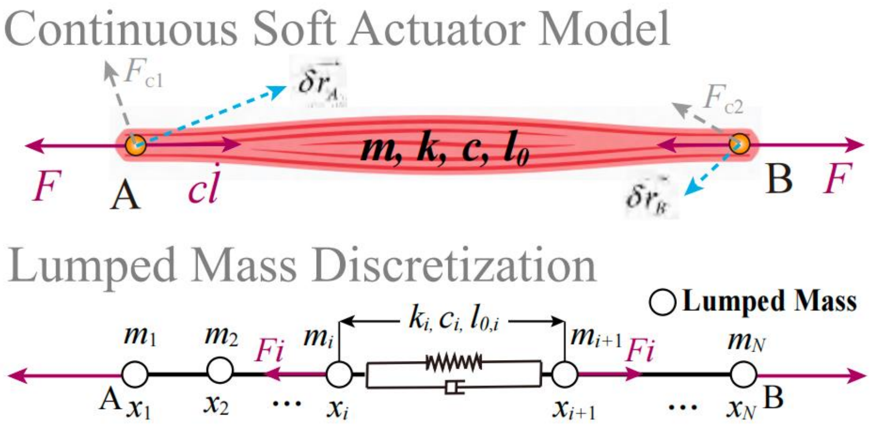
k：刚度
c：阻尼
l_0：原长
l：驱动器长度
F：驱动力
$\delta\vec{r}$：虚位移
F_c：约束力

利用第二类Lagrange方程推导整机动力学方程：

$$\left(\frac{d}{dt}\frac{\partial}{\partial \dot{\mathbf{q}}} - \frac{\partial}{\partial \mathbf{q}}\right)(L_{\mathrm{EA}} + L_{\mathrm{other}}) = \mathbf{Q}_{\mathrm{EA}} + \mathbf{Q}_{\mathrm{other}} \qquad (1)$$

- 将系统的 Lagrange 量（拉格朗日量）和广义力部分进行分解为与所研究弹性体驱动器相关（EA）和无关的部分（other）。若系统的能量和广义力形式不变，则系统的动力学模型保持不变

3. 研究内容| 离散化模型及其能量等效分析



直线形人工肌肉及假想等效模型的力学简图
（省略重力、弹性力等有势力）

目标：希望找到等效的多刚体表示

重力势能、动能形式推导

$\hat{\cdot}$：离散模型对应物理量

$$V_g = \frac{1}{2} mg\vec{k} \cdot (\vec{r}_A + \vec{r}_B) \quad (2)$$

$$= \frac{1}{2}\vec{k} \cdot \left[\frac{1}{6} m\vec{r}_A + \frac{2}{3} m\left(\frac{\vec{r}_A + \vec{r}_B}{2}\right) + \frac{1}{6} m\vec{r}_B\right] \quad (3)$$

$$\hat{V}_g = \sum_{i=1}^{N} m_i g\vec{k} \cdot \vec{x}_i = \sum_{i=1}^{N} m_i g\vec{k} \cdot \left[\left(1 - \frac{x_i}{l}\right)\vec{r}_A + \frac{x_i}{l}\vec{r}_B\right] \quad (4)$$

$$T = \int_0^l \frac{1}{2} v^2(x) \frac{m}{l} dx = \frac{1}{6} m(\vec{v}_A \cdot \vec{v}_B + v_A^2 + v_B^2) \quad (5)$$

$$= \frac{1}{2}\left[\frac{1}{6} m v_A^2 + \frac{2}{3} m\left(\frac{\vec{v}_A + \vec{v}_B}{2}\right)^2 + \frac{1}{6} m v_B^2\right] \quad (6)$$

$$\hat{T} = \sum_{i=1}^{N} \frac{1}{2} m_i \left\| \frac{x_i}{l}\vec{v}_B + \frac{l - x_i}{l}\vec{v}_A \right\|^2. \quad (7)$$

重力势能与动能等效的等价条件

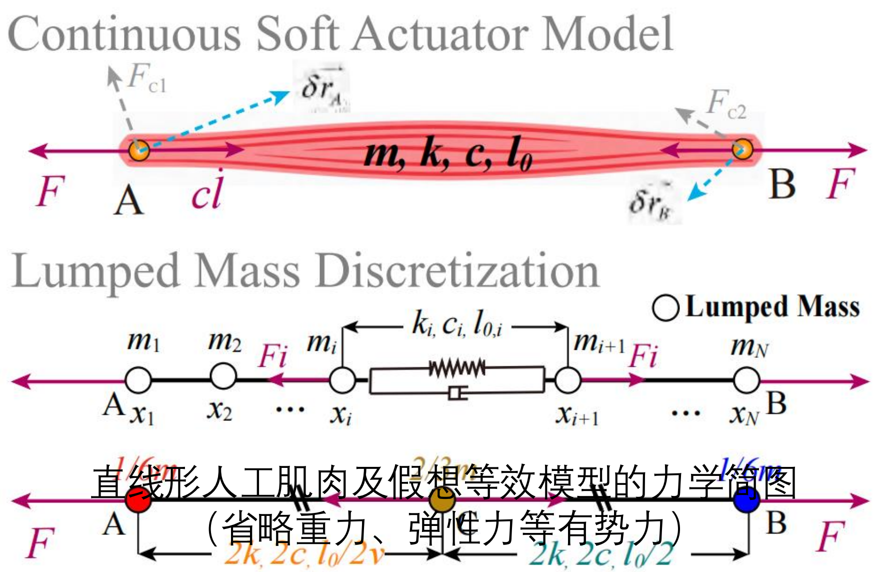
$\hat{x}_i = x_i/l, \ \hat{m}_i = m_i/m$

$$\left.\begin{matrix} \hat{T} = T \\ \hat{V}_g = V_g \end{matrix}\right\} \Longleftrightarrow \begin{bmatrix} 1 & \cdots & 1 \\ \hat{x}_1 & \cdots & \hat{x}_N \\ \hat{x}_1^2 & \cdots & \hat{x}_N^2 \end{bmatrix} \cdot \begin{bmatrix} \hat{m}_1 \\ \vdots \\ \hat{m}_N \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ 1/3 \end{bmatrix}. \quad (8)$$

动力学模型等效的必要条件是重力势能和动能部分对应相等，对等式（8）分类讨论

- 若 $N < 3$，方程无解
- 若 $N = 3$，方程在（$\hat{x}_1, \hat{x}_2, \hat{x}_3$互异）的情况下一定有解，
  - $\hat{x}_1, \hat{x}_3 = 0, 1$
  - $\hat{x}_2 = 1/2$ 时，约束对称性最好，同时若假设存在误差，则在插值意义下精度最高，由辛普森公式（Simpson）保证

3. 研究内容| 离散化模型及其能量等效分析



直线形人工肌肉及假想等效模型的力学简图

（省略重力、弹性力等有势力）

多刚体等效模型及其参数

参数构造

$$\begin{cases} F_1 = F_2 = F, \quad c_1 = c_2 = 2c, \\ k_1 = k_2 = 2k, \quad l_{10} = l_{20} = l_0/2. \end{cases} \tag{9}$$

弹性势能和广义力等效

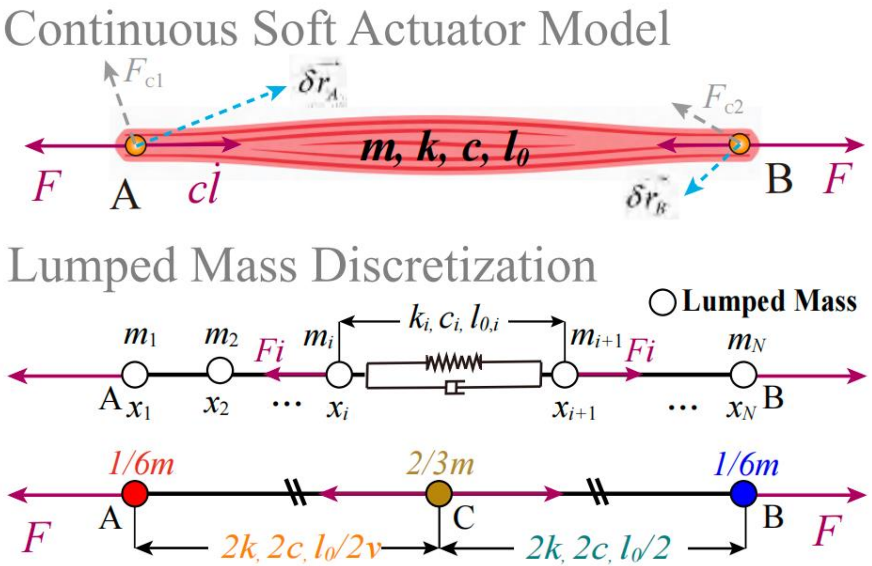
$$\begin{aligned} \hat{V}_e &= \frac{1}{2}k_1^{eq}(l/2 - l_{10})^2 + \frac{1}{2}k_1^{eq}(l/2 - l_{10})^2 \\ &= 2 \times \frac{1}{2} \times 2k(l - l_0)^2 = \frac{1}{2}k\left[(l - l_0)/2\right]^2 = V_e \end{aligned} \tag{10}$$

$$\begin{aligned} \delta\hat{W} &= (F_1 - c_1\dot{l}/2)\vec{e}_{BC} \cdot \delta\vec{r}_{BC} + (F_2 - c_2\dot{l}/2)\vec{e}_{CA} \cdot \delta\vec{r}_{CA} \\ &= (F - c\dot{l})\vec{e}_{BA} \cdot \delta\vec{r}_A + (F - c\dot{l})\vec{e}_{AB} \cdot \delta\vec{r}_B = \delta W \end{aligned} \tag{11}$$

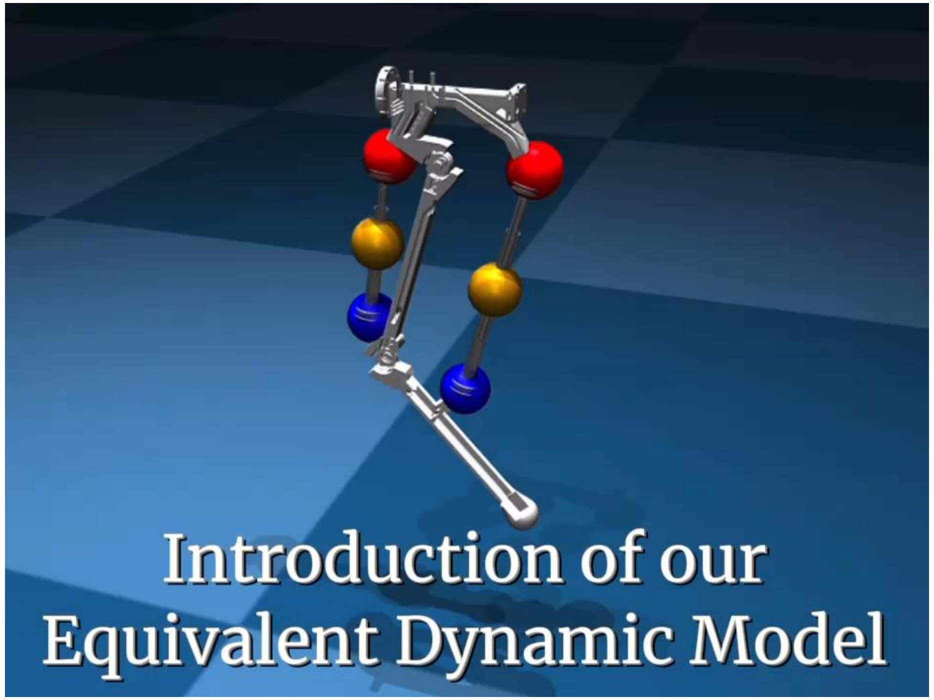
最终结论-系统动力学等效条件

- 等效模型由三个质点组成连杆部分，两个直线驱动器连接其中
- 等效模型的离散质量分布为原驱动器的 1/6，2/3，1/6
- 等效模型的直线驱动器（理想，无质量）刚度、阻尼为原系数的两倍，原长为原驱动器原长的一半
- 等效模型驱动过程中，两个直线驱动器部分的驱动力和长度始终保持一致。该条件通过约束满足

3. 研究内容| 离散化模型及其能量等效分析



多刚体等效模型及其参数



等效模型及仿真案例演示——气动驱动的机器人足

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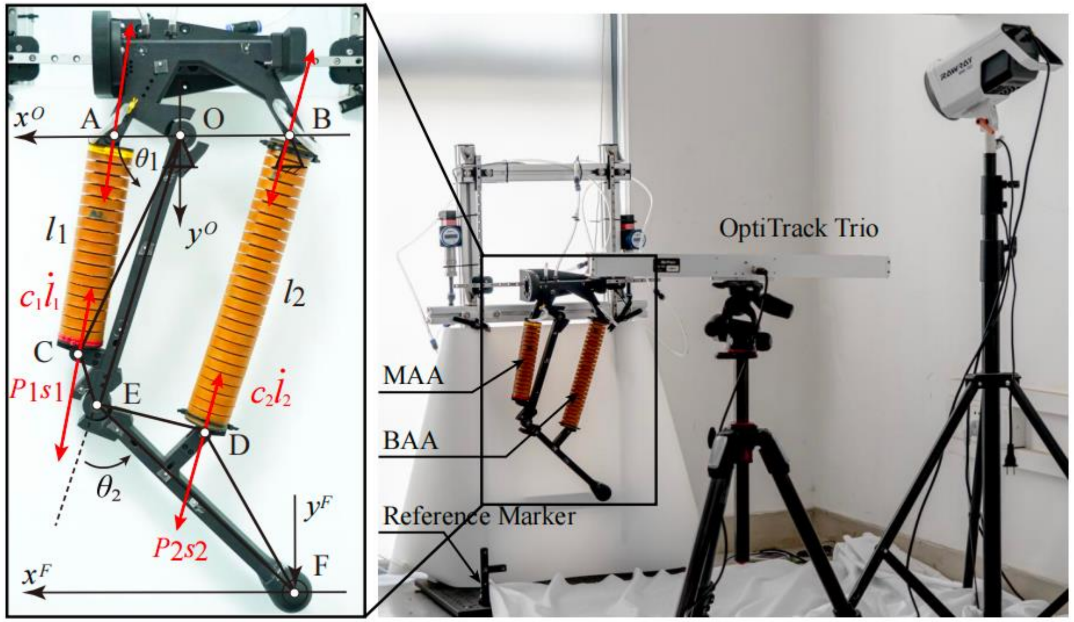
3. 研究内容| 仿真实验验证

理论动力学模型及仿真



(B) Coordinate Frame and Physical Implementation of Experimental Platform

利用拉格朗日方法，对刚柔耦合系统进行动力学建模，推导出两种状态下的理论动力学方程，并通过Matlab ODE45实现动力学仿真

各部分能量

驱动器1 (MAA)

$$V_{a1} = -\frac{1}{2}m_3 g(A_y + C_y) + \frac{1}{2}k_3(l_1 - l_{10})^2$$

$$T_{a1} = \frac{1}{6}m_3\big(v_{1C}(1)\cdot v_{1C}(1) + v_{1C}(2)\cdot v_{1C}(2) + v_{2C}(1)\cdot v_{2A}(1) + v_{2C}(2)\cdot v_{2A}(2) + v_{2A}(1)\cdot v_{2A}(1) + v_{2A}(2)\cdot v_{2A}(2)\big)$$

驱动器2 (BAA)

$$V_{a2} = -\frac{1}{2}m_4 g(B_y + D_y) + \frac{1}{2}k_4(l_2 - l_{20})^2$$

$$T_{a2} = \frac{1}{6}m_4\big(v_{2B}(1)\cdot v_{2B}(1) + v_{2B}(2)\cdot v_{2B}(2) + v_{2D}(1)\cdot v_{2D}(1) + v_{2D}(2)\cdot v_{2D}(2) + v_{2B}(1)\cdot v_{2D}(1) + v_{2B}(2)\cdot v_{2D}(2)\big)$$

基座

$$T_M = \frac{1}{2}M\left(\frac{d}{dt}(O_y)^2\right)$$

$$V_M = -Mg\cdot O_y$$

大腿

$$V_1 = -\frac{1}{2}m_1 g(O_y + E_y)$$

$$T_1 = \frac{1}{2}I_1\left(\frac{d}{dt}\theta_1\right)^2 + \frac{1}{2}m_1\cdot\frac{1}{4}\left(\left(\frac{d}{dt}(E_x + O_x)\right)^2 + \left(\frac{d}{dt}(E_y + O_y)\right)^2\right)$$

小腿

$$V_2 = -\frac{1}{2}m_2 g(F_y + E_y)$$

$$T_2 = \frac{1}{2}I_2\left(\frac{d}{dt}(\theta_1 + \theta_2)\right)^2 + \frac{1}{2}m_2\cdot\frac{1}{4}\left(\left(\frac{d}{dt}(E_x + F_x)\right)^2 + \left(\frac{d}{dt}(E_y + F_y)\right)^2\right)$$

Lagrange量

$$T = T_M + T_1 + T_2 + T_{a1} + T_{a2}$$

$$V = V_M + V_1 + V_2 + V_{a1} + V_{a2}$$

$$L = T - V$$

系统动力学方程

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q_q$$

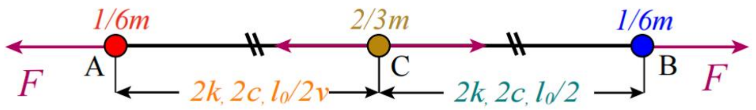
一阶常微分方程（ODE）形式——Matlab ODE45做数值仿真

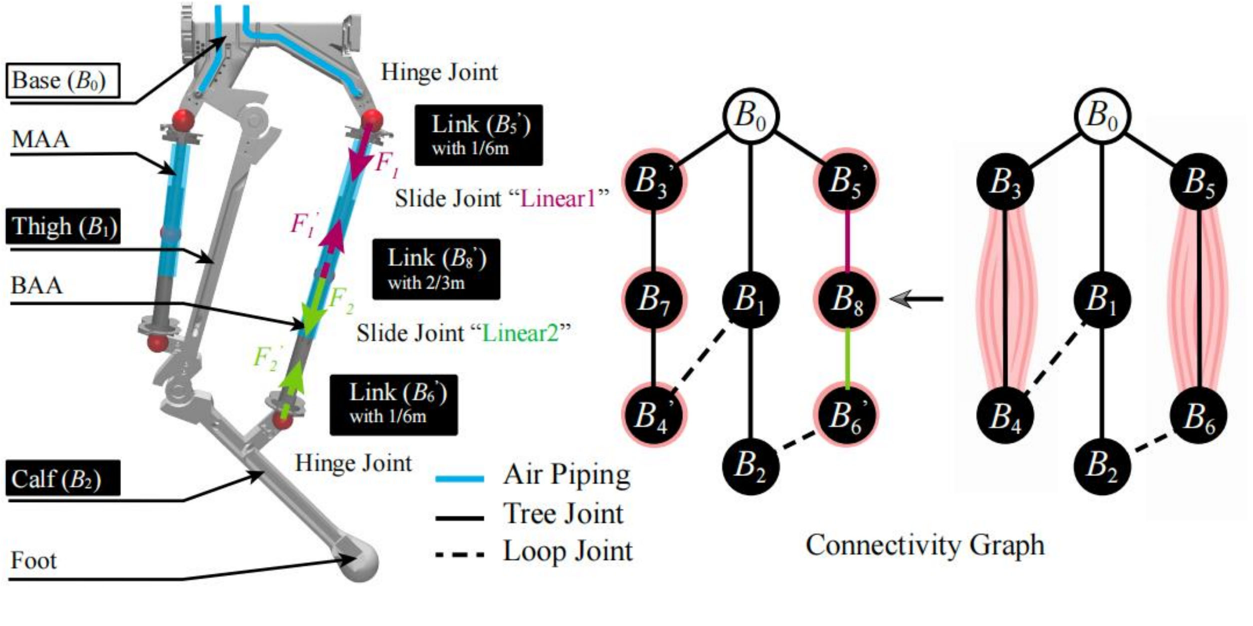
Stance/Swing Phase

$$\frac{\mathrm{d}\begin{bmatrix}\theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2\end{bmatrix}^{\mathrm{T}}}{\mathrm{d}t} = f(\begin{bmatrix}\theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2\end{bmatrix}^{\mathrm{T}}, F, m, k, c, l).$$

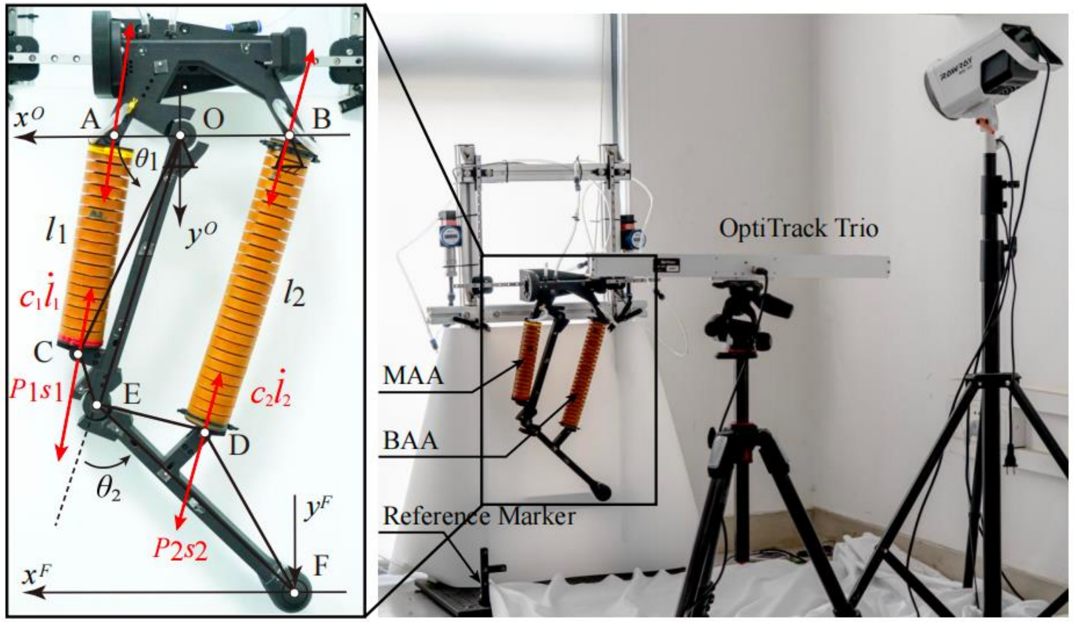
3. 研究内容| 仿真实验验证

□ 等效动力学模型及仿真的软件实现





(A) Mujoco Implementation of Equivalent Model of Robotic Leg

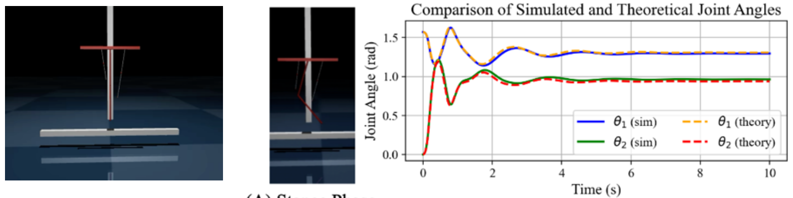


(B) Coordinate Frame and Physical Implementation of Experimental Platform

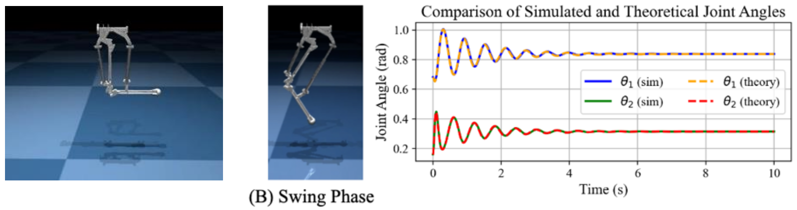
- □ 拆并联结构为串联结构，从URDF格式编译为Mujoco-MJCF形式
- □ 基于等效方法构建肌肉等效离散质量分布，增加直线驱动器，施加位移约束
- □ 通过约束恢复并联结构，驱动时保持力约束
- □ 在刚体仿真器 Mujoco 中直接进行仿真和数据采集，无需进行具体的动力学推导

3. 研究内容| 实验验证

□ 静态和动态特征验证



(A) Stance Phase



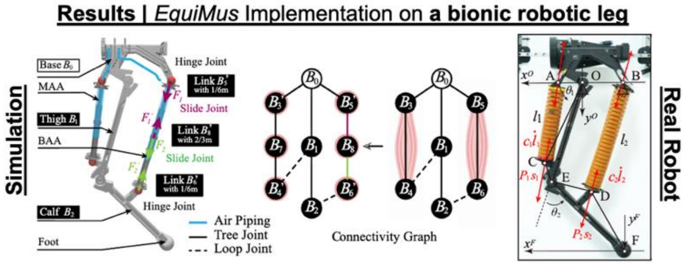
(B) Swing Phase

理论模型和等效模型在仿真中的阶跃响应误差

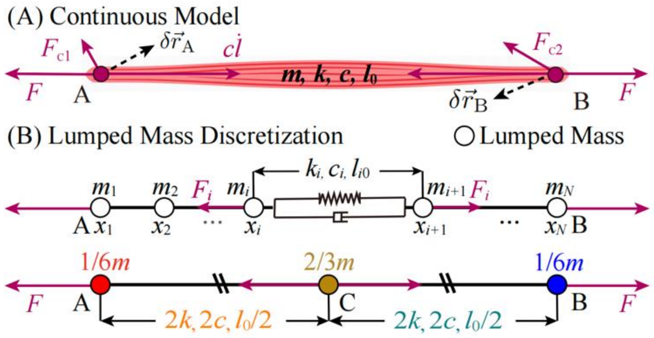
Metric	Static θ_1 / θ_2	Dynamic-Swing θ_1 / θ_2	Dynamic-Stance θ_1 / θ_2
RMSE(rad)	0.00096 / 0.00574	0.00094 / 0.00586	0.01693 / 0.03000
MAE(rad)	0.00081 / 0.00547	0.00079 / 0.00556	0.01546 / 0.02829
MaxAE(rad)	0.00369 / 0.01452	0.00379 / 0.01559	0.04673 / 0.08805

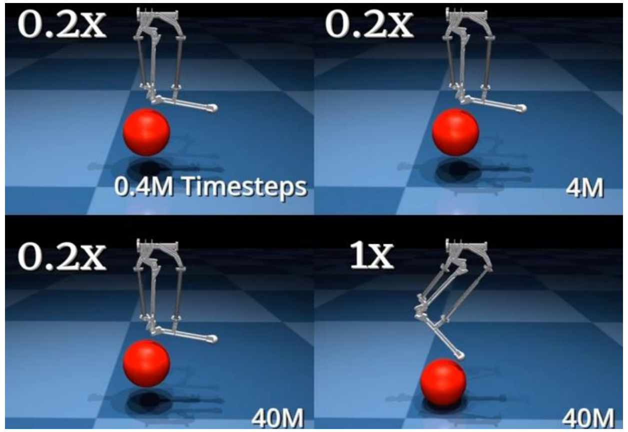
RMSE ~ 5.56°和 7.84°，部分误差来源于阀门的迟滞效应、静摩擦力等原因

Takeaways



$$\left(\frac{d}{dt}\frac{\partial}{\partial \dot{\mathbf{q}}} - \frac{\partial}{\partial \mathbf{q}}\right)(L_{\mathrm{EA}}+L_{\mathrm{other}}) = \mathbf{Q}_{\mathrm{EA}}+\mathbf{Q}_{\mathrm{other}}$$





We proposed ***EquiMus***, an unified energy-equivalent dynamics and simulation algorithm for the rigid-soft musculoskeletal robots with linear elastic actuators. The method captures **dynamic mass redistribution, supports loop-closure constraints** in **MuJoCo**, and remains real-time capable.

<u>Bridge Soft and Rigid</u>

Energy-consistent discretization preserves the overall system dynamics.

<u>A light-weight implementation</u>

The soft actuator is **discretized into a 3–2–1 configuration**:

3 Mass Points: capture inertia distribution.

2 Linear Actuators: represent elastic and damping behavior.

1 Constraint: enforces equal elongation, preserving symmetry and energy consistency.

<u>Downstream Applicatoin</u>

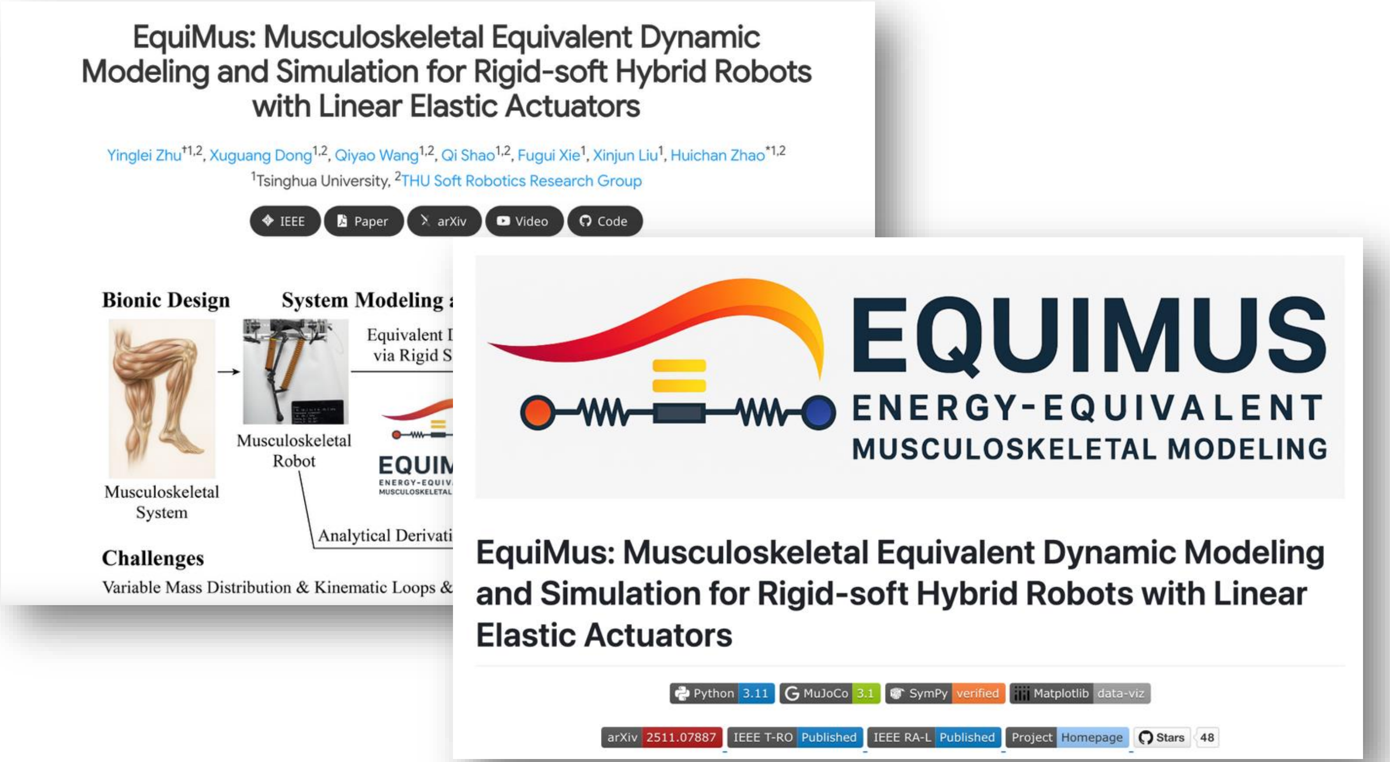
Example including **PID auto-tuning**, **model-based control**, and **RL tasks**

Thanks

🎶 Design Philosophy

[EN] EquiMus fakes how nature uses energy to control motion — through an energy-equivalent formulation that turns physics intuition into simulation reality. That is the “fake it until you make it” philosophy in the energy domain.

[CN] 假之以能量，得之于运动。以伪成真，以虚造实。自然之道，人之所驭。



Webpage & GitHub

Wechat

EquiMus is open-source at https://github.com/fly-pigTH/EquiMus